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SEMI-STRONG COLOR PARTITION OF A GRAPH

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ABSTRACT

Claude Berge introduced the concept of strong stable sets in a graph. A subset S of a graph $G = (V, E)$ is a strong stable set if $|N[v] \cap S| \leq 1$ for every $v \in V(G)$. Relaxing this condition Prof.E. Sampath kumar introduced semi-strong sets in graphs as those sets for which $|N(v) \cap S| \leq 1$ for every $v \in V(G)$.Resolvability is a well-studied concept. Combining these two, resolving semi-strong color partition is defined and studied in this paper.

Classification: 05C15, 05C70

Keywords: *Resolving semi-strong color partition.*

I. INTRODUCTION

A subset S of a graph G = (V, E) is called a semi-strong set if $|N[v] \cap S| \leq 1$ for every $v \in V(G)$.

A subset S = $\{x_1, x_2, x_3, \ldots\ldots, x_k\}$ of a connected graph G is called a resolving set if the code C(u : S)=(d(u, x_1), $d(u, x_1), \ldots\ldots, d(u, x_1)$) is different for different u. A partition of V(G) into subsets where each subset considered is a resolving semi-strong set. The Minimum cardinality of such a partition denoted by $\chi_{spd}(G)$ is found out for some well-known graphs. Further, graphs with $\chi_{spd}(G)$=2,$\chi_{spd}(G)$= n are determined.

II. RESOLVING SEMI- STRONG COLOR PARTITION

Definition 1.1. Let G be a finite, simple, connected, undirected graph. A partition $\Pi = \{V_1, V_2, \ldots, V_k\}$ is called a resolving semi strong color partition if Π is a semi-strong color partition and the k-vector $(v|\Pi)=(d(v,v_1),d(v,v_2),\ldots,d(v,v_k))$ is distinct for different v in V (G). The minimum cardinality of a resolving semi-strong color partition of G is called semi-strong color class partition dimension of G and is denoted byχ_{spd}(G). The trivial partition namely$\{\{v_1\},\{v_2\} \ldots,\{v_k\}\}$ where $V(G)=\{v_1,v_2,.v_k\}$ is a resolving semi-strong color class partition of G.

Remark 1.2. (i)$\chi_s(G) \leq \chi_{spd}(G)$.
(ii) $pd(G) \leq \chi_{spd}(G)$

Example 1.3. Let G be the graph given in Fig.1.1:$\chi_s(G) = 5$.Therefore $\chi_{spd}(G)$=5.

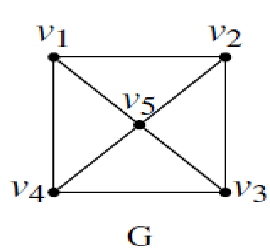

Figure 1.1

Example1.4. Let $G = P_n$. Let $V(P_n) = \{u_1, u_2, \ldots, u_n\}$, $n \geq 3$.Let $\Pi = \{\{u_1\}, \{u_2, u_3, u_6, u_7, \ldots\}, \{u_4\}, \{u_5\}, \{u_8\}, \ldots\}$. Π is a minimum resolving semi-strong color partition of P_n. Therefore $\chi_{spd}(P_n) = 3$, $n \geq 4$. when n = 1, 2, 3then, $\chi_{spd}(P_1) = 1$, $\chi_{spd}(P_2) = 2$, $\chi_{spd}(P_3) = 2$.

χ_{spd}(G) for some well-known Graphs

Proposition:

1. $\chi_{spd}(K_n) = n$.
2. $\chi_{spd}(K_{1,n}) = n$.
3. $\chi_{spd}(K_{m,n}) = \begin{cases} n & \text{if } m < n \\ n+1 & \text{if } m = n \end{cases}$
4. $\chi_{spd}(W_n) = n$
5. $\chi_{spd}(P) = 5$ where P is the Petersen graph.

Proof :Let $V(P) = \{v_1, v_2, v_3, v_4, v_5, w_1, w_2, w_3, w_4, w_5\}$. Consider the Petersen graph given in Figure 1.2.Here pd(P) = 4, but $\chi_s(P) = 5$. Also $\chi_{spd}(P) = 5$.

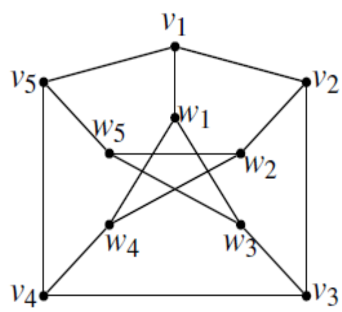

Figure 1.2

6. $\chi_{spd}(P_n) = \begin{cases} 3 & \text{if } n \geq 4 \\ 2 & \text{if } n = 2, 3 \\ 1 & \text{if } n = 1 \end{cases}$

7. $\chi_{spd}(C_n) = 3$ for every $n \geq 3$

Proof: **Case 1**: Let $n \equiv 0 \pmod 4$. Let n = 4k.
Let $V(C_n) = \{\{u_1, u_2, \ldots, u_{4k}\}$. Let $\Pi = \{\{u_{4k-2}, u_{4k-1}\}, \{u_1, u_2, u_5, u_6, u_9, u_{10}, \ldots, u_{4k-7}, u_{4k-6}, u_{4k-3}\}, \{u_3, u_4, u_7, u_8, \ldots, u_{4k-5}, u_{4k-4}, u_{4k}\}\}$. Then Π is a resolving semi-strong color partition of C_n. Therefore $\chi_{spd}C_n) \leq 3$. Suppose $\chi_{spd}(C_n) = 1$. Then n = 1, a contradiction. If $\chi_{spd}(C_n) \neq 2$,since$\chi_{spd}(G) = 2$ if and only if G =P_2 orP_3. Therefore $\chi_{spd}(C_n) = 3$ when $n \equiv 0(\bmod 4)$.

Case 2: Let $n \equiv 1 \pmod 4$. Let n = 4k + 1.
Let $V(C_n) = \{u_1, u_2, \ldots, u_{4k+1}\}$.Let $\Pi = \{\{u_{4k+1}\}, \{u_1, u_2, u_5, u_6, \ldots, u_{4k-3}, u_{4k-2}\}, \{u_3, u_4, u_7, u_8, \ldots, u_{4k-1}, u_{4k}\}$. Then it can be easily verified that Π is a resolving semi-strong color partition of C_n. Therefore, $\chi_{spd}(C_n) \leq 3$. But $\chi_{spd}(C_n) \geq 3$, since $\chi_{spd}(G) = 2$ if and only if G = P_2or P_3 and $\chi_{spd}(G) = 1$ if and only if G = K_1.
Therefore , $\chi_{spd}(C_n) = 3$when $n \equiv 1 \pmod 4$.

Case 3: Let $n \equiv 2 \pmod 4$. Let n = 4k + 2.
Let $V(C_n) = \{u_1, u_2, \ldots, u_{4k+2}\}$.

Let $\Pi = \{\{u_{4k+1}, u_{4k+2}\}, \{u_1, u_2, u_5, u_6, \ldots, u_{4k-3}, u_{4k-2}\}, \{u_3, u_4, u_7, u_8, \ldots, u_{4k-1}, u_{4k}\}$. Then it can be easily verified that Π is a resolving semi-strong color partition of C_n. Therefore, $\chi_{spd}(C_n) \leq 3$. But $\chi_{spd}(C_n) \geq 3$, since $\chi_{spd}(G) = 2$ if and only if $G = P_2$or P_3 and $\chi_{spd}(G) = 1$ if and only if $G = K_1$.
Therefore ,$\chi_{spd}(C_n) = 3$when $n \equiv 2(\text{mod } 4)$.

Case 4: Let $n \equiv 3(\text{mod } 4)$. Let $n = 4k + 3$.
Let $V (C_n) = \{u_1, u_2, \ldots, u_{4k+3}\}$.
Let $\Pi = \{\{u_{4k+2}\}, \{u_1, u_2, u_5, u_6, \ldots, u_{4k-3}, u_{4k-2}, u_{4k+1}\}, \{u_3, u_4, u_7, u_8, \ldots, u_{4k-1}, u_{4k}, u_{4k+3}\}$. Then it can be easily verified that Π is a resolving semi-strong color partition of C_n. Therefore $\chi_{spd}(C_n) \leq 3$. But $\chi_{spd}(C_n) \geq 3$.
Therefore ,$\chi_{spd}(C_n) = 3$ when $n \equiv 3(\text{mod } 4)$. e
Hence $\chi_{spd}(C_n) = 3$ for every $n \geq 3$.

Theorem 1.5. $\chi_{spd}(G) = 2$ if and only if $G = P_2$or P_3.

Proof: Let $V (G) = \{u_1, u_2, \ldots, u_n\}$. Let $\chi_{spd}(G) = 2$. Let $\Pi = \{V_1, V_2\}$ be a semi strong color class partition of G. Then there exist vertices $u_i \in V_1$, $u_j \in V_2$ such that u_i and u_j are adjacent (since G is connected). Suppose u_i is adjacent with v_{j1} and v_{j2} in V_2. $r (v_{j1} |\Pi) = (1, 0)$, $r (v_{j2} |\Pi) = (1, 0)$, a contradiction. Therefore, u_i is a unique vertex in V_1 that is adjacent to a vertex in V_2 and u_j is the unique vertex in V_2 that is adjacent to a vertex in V_1. Suppose $|V_1| \geq 2$. Let $u_{i1} \in V_1$. Suppose u_{i1} is not adjacent with any vertex of V_1.Then $r(u_i|\Pi)=(0,1)$ and $r (u_{i1} |\Pi) = (0, 1)$, a contradiction. Therefore, u_{i1} is adjacent with at least one vertex of V_1 and not adjacent with any vertex of V_2. u_i is adjacent with at most one vertex in V_1. For if u_i is adjacent with u_1,u_2 in V_1, then $r(u_1|\Pi)=(0,2)$ and $r(u_2|\Pi)=(0,2)$,a contradiction. Therefore, u_i is adjacent with exactly one vertex in V_1. Let w be a unique vertex in V_1 which is adjacent with u_i. If w is adjacent with another vertex in V_1, then V_1 is not semi strong. Therefore, w is adjacent with only u_i. Therefore, u_iw is a component of V_1. Further w is not adjacent with any vertex of V_2. For if w is adjacent with $w_1 \in V_2$, then $r(v_j|\Pi) = r(w_1|\Pi)$, a contradiction. If V_1 contains a third vertex x distinct from u_i and w. Then x is adjacent to some vertex of V_2, a contradiction, since that vertex w_1 and v_j have the same coordinate. Therefore $|V_1| = 2$. If $|V_2| \geq 2$, then proceeding as before$|V_2| = 2$ and $G = P_4$. But $\chi_{spd}(P_4) = 3$, a contradiction. Therefore $|V_2| \leq 1$.Hence $|V_2| = 1$ we get $G = P_3$. If $|V_1| = 1$, then $G = P_2$ or P_3. Therefore $\chi_{spd}(G) = 2$ if and only if $G = P_2$ or P_3.

Theorem 1.6. Let G be a graph with full degree vertex, say u. Then $\chi_{spd}(G) = n$ if and only if the subgraph induced by a vertex of G other than u has no isolates.

Proof: Suppose G has a full degree vertex say u. Let $v_1, v_2, \ldots, v_{n-1}$ are the vertices of G adjacent with u. Then no two vertices v_i, v_j, $i \neq j$ belong to the same color class of a resolving semi strong color partition of G.

Case1: Suppose the subgraph induced by vertices of G other than u has an isolate say v_1.
Let $\Pi = \{\{u, v_1\}, \{v_2\}, \ldots, \{v_{n-1}\}\}$. By a hypothesis, v_i is adjacent with u for every i, v_i is not adjacent with v_1 for any i. Clearly u, v_1 are resolved by any v_i, $(i \geq 2)$. Therefore ,$\chi_{spd}(G) = n - 1$.

Case 2: Suppose the subgraph induced by vertices of G other than u has no isolate.
Then Π is not a semi strong color partition of G.Therefore,$\Pi_1=\{\{u\},\{v_1\},\ldots,\{v_{n-1}\}\}$ is a resolving semi strong color partition of G and it is minimum. That is $\chi_{spd}(G) =n$.

Theorem 1.7. Let G be a connected graph. $\chi_{spd}(G) = n$ if and only if $N (G) =K_n$.

Proof: Let $\chi_{spd}(G) = n$. Let $V (G) = \{u_1, u_2, \ldots, u_n\}$. Suppose $diam(G) = k \geq 3$.Let $u = u_1, u_2, \ldots, u_{k+1} = v$ be a diametrical path in G. Let $\Pi = \{\{u, v\}, \{V_2\}, \ldots,\{V_{n-1}\}\}\{\{u, v\}, \{V_2\}, \ldots, \{V_{n-1}\}\}$ where $V_2, \ldots, V_{n-1}$ are singletons .u and v are resolved by $\{u_2\}$. Then Π is are solving semi strong color partition of G. Therefore, $\chi_{spd}(G) \leq n-1$, a contradiction. Therefore, $diam(G) \leq 2$. Suppose u_1 and u_2 are adjacent and u_1u_2 is not the edge of a triangle. Let $\Pi_1=\{\{u_1,u_2\},V_2,\ldots,V_{n-1}\}$ where $V_2,\ldots,V_{n-1}$.$u_1$and $u_2$are resolved by $\{u_3\}$ where u_1 is adjacent with $u_3$and u_2 is not adjacent with u_3. Then Π_1 is a resolving semi strong color partition of G, a contradiction.

Let $|V(G)| \geq 4$. If u_1 and u_2 are adjacent. Then u_1u_2 is an edge of triangle. Therefore, $N(G) = K_n$.
Suppose $|V(G)| = 3$. Then $G = P_3$ or K_3. $\chi_{spd}(P_3) = 2 < 3$. Therefore, $G = K_3$. Therefore, $N(G) = K_n$.
The converse is obvious.

Theorem 1.8. For $m \geq 3$, $\chi_{spd}(K_{a_1,a_2,\ldots,a_m}) = a_1 + a_2 + \ldots + a_m$.

Proof: $\chi_s(K_{a_1,a_2,\ldots,a_m}) = a_1+a_2+\ldots+a_m$. Further $\chi_s(K_{a_1,a_2,\ldots,a_m}) \leq \chi_{spd}(K_{a_1,a_2,\ldots,a_m})$.
Therefore, $\chi_{spd}(K_{a_1,a_2,\ldots,a_m}) = a_1 + a_2 + \ldots + a_m$.

Theorem 1.9. Let $G = K_m(a_1, a_2, \ldots, a_m)$. Then $\chi_{spd}(G) = m + \max\{a_i\}, 1 \leq i \leq m$.

Proof: Let $G = K_m(a_1, a_2, \ldots, a_m)$. Let $V(G) = \{u_1, u_2, \ldots, u_m, u_{1,1}, u_{1,2}, \ldots, u_{1,a_1}, \ldots, u_{m,1}, u_{m,2}, \ldots, u_{m,a_m}\}$ where $V(K_m) = \{u_1, u_2, \ldots, u_m\}$. Let $\Pi = \{\{u_1\},\{u_2\},\ldots,\{u_m\},\{u_{1,1},u_{2,1},\ldots,u_{m,1}\},\ldots,\}$. Then Π is a resolving semi strong color partition of G. Therefore, $\chi_{spd}(G) \leq m + \max\{a_i\}$, $1 \leq i \leq m$. Since $\chi_s(G) \leq \chi_{spd}(G)$ and $\chi_s(G) = m + \max\{a_i\}$, $1 \leq i \leq m$. Therefore, $\chi_{spd}(G) = m + \max\{a_i\}$, $1 \leq i \leq m$.

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